

3/17/14 "I make the most of all that comes and the least of all that goes."-Sara Teasdale

HW: Exercises #1, 3, 5, 8, 11, 12
Test #2 on Friday 3/21

AIM: How do we perform operations on matrices?

Warm Up:

Two matrices are equal if they have the same order $m \times n$ and their corresponding entries are equal.

1. Solve for a_{11} , a_{12} , a_{21} , and a_{22} in the following matrix equation.

$$a_{11} = 2$$

$$a_{12} = -1$$

$$a_{21} = -3$$

$$a_{22} = 0$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

Matrix Addition

You can add two matrices (of the same order) by adding their corresponding entries. The sum of two matrices of different orders is undefined.

$$2. \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1+1 & 2+3 \\ 0+(-1) & 1+2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$$

Additive
Identity

$$3. \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$4. \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5. \text{ The sum of } A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix} \text{ is}$$

$3 \times 3 \qquad 3 \times 2$

Undefined
(not the same order)

Scalar Multiplication

In work with matrices, numbers are usually referred to as scalars. For our purposes, scalars will always be real numbers. You can multiply a matrix A by a scalar c by multiplying each entry in A by c .

The symbol $-A$ represents the scalar product $(-1)A$. Moreover, if A and B are of the same order, $A - B$ represents the sum of A and $(-1)B$. That is,

$$A - B = A + (-1)B \quad (\text{Subtraction of matrices})$$

6. For the following matrices, find (a) $3A$
 (b) $-B$
 (c) $3A - B$

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

$$\text{a) } 3 \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix} \quad \text{3A}$$

$$\text{b) } -1 \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$\text{c) } 3A - B = 3A + -B$$

$$\begin{bmatrix} 6 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 12 \\ -10 & 4 & -6 \\ 7 & 0 & 4 \end{bmatrix}$$

$3A - B$

Properties of Matrix Addition and Scalar Multiplication

Let A , B , and C be $m \times n$ and let c and d be scalars.

1. $A + B = B + A$ Commutative Prop of Matrix Add.

2. $A + (B + C) = (A + B) + C$ Associative Prop of Matrix Add.

3. $(cd)A = c(dA)$ Associative Prop of Scalar Mult.

4. $IA = A$ Identity Property of Mult.

5. $c(A + B) = cA + cB$ Distributive Prop.

6. $(c + d)A = cA + dA$ Multiplication.

7. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \checkmark$

8. Solve for X in the equation $3X + A = B$, where

$$\begin{array}{r} 3X + A = B \\ -A \quad -A \\ \hline 3X = B - A \\ \frac{3X}{3} = \frac{B - A}{3} \end{array}$$

$$X = \frac{1}{3}(B - A)$$

$$X = \frac{1}{3} \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{4}{3} & 2 \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$\begin{array}{c} \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix} \xleftarrow{B} \quad \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \xleftarrow{A} \\ \hline \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix} \\ B - A \end{array}$$

$$12) \quad A = \begin{bmatrix} -2 & -1 \\ 1 & 6 \\ 3 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$\cancel{2X} = \frac{2A - B}{2}$$

$$X = \frac{1}{2}(2A - B)$$

$$X = \frac{1}{2} \begin{bmatrix} -4 & -5 \\ 0 & 0 \\ 10 & -7 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -\frac{5}{2} \\ 0 & 0 \\ 5 & -\frac{7}{2} \end{bmatrix}$$

$$\begin{matrix} 2A & - & B \\ \begin{bmatrix} -4 & -2 \\ 2 & 0 \\ 6 & -8 \end{bmatrix} & - & \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} \\ \begin{bmatrix} -4 & -5 \\ 0 & 0 \\ 10 & -7 \end{bmatrix} \end{matrix}$$

Matrix Multiplication

To find the entries of the product, multiply each row of A by each column of B . Note that the number of columns of A must be equal to the number of rows of B .

⊛ The # of columns in the first matrix MUST equal the # of rows in the second.

9. Find the product AB where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1(2)+3(-4) & -1(2)+3(1) \\ 4(-2)+2(-1) & 4(2)+2(1) \\ 5(-2)+0(-4) & 5(2)+0(1) \end{bmatrix} = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -10 & 10 \end{bmatrix}$$

$$10. \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -2+0-3 & 4+0+3 & 2+0-3 \\ -4-1+2 & 8+0-2 & 4+0+2 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 7 & -1 \\ -3 & 6 & 6 \end{bmatrix}$$

$$11. \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{Identity}$$

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

$$12. \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow \text{Identity } 2 \times 2 \text{ Matrix}$$

$$13. \begin{bmatrix} 1 & -2 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

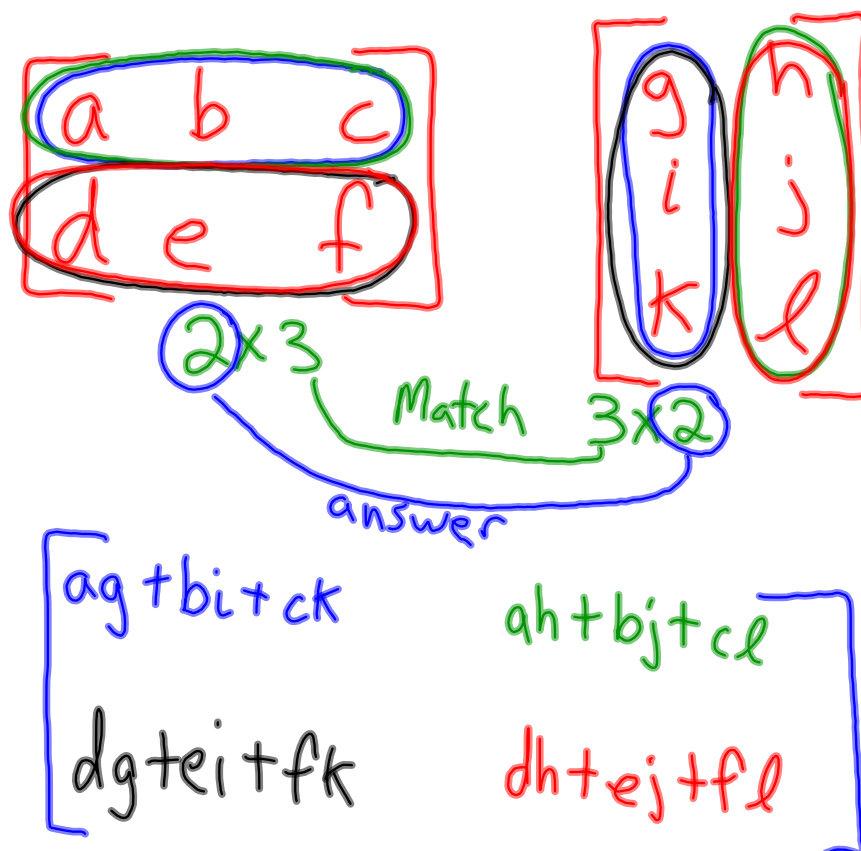
$$\begin{bmatrix} 2+2-3 \\ 2+2-3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$14. \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$$

$$15. \text{ Find the product of } AB. \text{ If } A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$$

NO Solution!

I can do Math!



HW: 16, 19, 21, 23, 25

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2 \times 3} \quad \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} ag + bi + ck & ah + bj + cl \\ dg + ei + fk & dh + ej + fl \end{bmatrix}$$

The $m \times n$ matrix that consists of 1's on its main diagonal and 0's elsewhere is called the **identity matrix of order n** and is denoted by:

 X

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

An identity matrix must be square.

16. Solve the matrix equation $AX = B$ for X , where

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 4 \\ 2 \end{bmatrix}.$$

Exercises
(courtesy of our textbook)

In Exercises 1–4, find x and y .

$$1. \begin{bmatrix} x & -2 \\ 7 & y \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ 7 & 22 \end{bmatrix}$$

$$2. \begin{bmatrix} -5 & x \\ y & 8 \end{bmatrix} = \begin{bmatrix} -5 & 13 \\ 12 & 8 \end{bmatrix}$$

$$3. \begin{bmatrix} 16 & 4 & 5 & 4 \\ -3 & 13 & 15 & 6 \\ 0 & 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 2x+1 & 4 \\ -3 & 13 & 15 & 3x \\ 0 & 2 & 3y-5 & 0 \end{bmatrix}$$

$$4. \begin{bmatrix} x+2 & 8 & -3 \\ 1 & 2y & 2x \\ 7 & -2 & y+2 \end{bmatrix} = \begin{bmatrix} 2x+6 & 8 & -3 \\ 1 & 18 & -8 \\ 7 & -2 & 11 \end{bmatrix}$$

In Exercises 5–10, find (a) $A + B$, (b) $A - B$, (c) $3A$, and (d) $3A - 2B$.

$$5. A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix}$$

$$7. A = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix}$$

$$8. A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix}$$

$$9. A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ 1 & 1 & -2 & 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & -1 & 1 & 0 \\ -3 & 4 & 9 & -6 & -7 \end{bmatrix}$$

$$10. A = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, \quad B = \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

In Exercises 11–14, solve for X given

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$11. X = 3A - 2B$$

$$12. 2X = 2A - B$$

$$13. 2X + 3A = B$$

$$14. 2A + 4B = -2X$$

In Exercises 15–20, find (a) AB , (b) BA , and, if possible, (c) A^2 . (Note: $A^2 = AA$.)

$$15. A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -1 & 8 \end{bmatrix}$$

$$16. A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$$

$$17. A = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$

$$18. A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$$

$$19. A = \begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix}$$

$$20. A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

In Exercises 21–28, find AB , if possible.

$$21. A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$$

$$22. A = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$$

$$23. A = \begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$

$$24. A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$25. A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$26. A = \begin{bmatrix} 10 \\ 12 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & -2 & 1 & 6 \end{bmatrix}$$

$$27. A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & -3 \\ 0 & 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & -11 & 4 \\ 8 & 16 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$28. A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 6 & 13 & 8 & -17 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}$$